Uncertainty analysis of a model of an energy distribution system with solar panel generation by Time-Varying Data Analysis, Monte Carlo Simulation and Fuzzy Interval Analysis

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ABSTRACT: The uncertainties in the model of an energy distribution system made of a solar panel, a storage energy system and loads (power demanded by the end-users) are investigated, treating the epistemic variables as possibilistic and the aleatory ones as probabilistic. In particular, time-varying probabilistic distributions of the solar irradiation and the power demanded by the end-users is inferred from historical data. Then a computational framework for the joint propagation of both types of uncertainty is applied through the model of the energy distribution system to compute the Expected Energy Not Supplied.

1 INTRODUCTION

Renewable energy is getting more and more important as a solution for the climate change concerns. However, it is affected by large uncertainty due to i) the intermittent nature of the energy source (the amount of energy daily available can vary a lot from one season to another at the same site) and ii) the possible unavailability of the unit when it is required to generate (Borges 2012); these two issues mine the reliability of the renewable energy. In this work, we deal with the first one, considering two types of uncertainty: randomness due to inherent variability in the system behavior (aleatory uncertainty) and imprecision due to lack of knowledge and information on the system (epistemic uncertainty) as typically distinguished in system risk analysis (Helton 2004). Recently, the co-existence of aleatory and epistemic uncertainties has been addressed in the reliability assessment of distributed generation systems, representing the aleatory variables as probabilistic and the epistemic ones as possibilistic, and a hybrid propagation approach has been introduced (Li & Zio 2012).

In the present paper, we analyse the aleatory and epistemic uncertainties of a model of an energy distribution system made of a solar panel, a storage energy system and loads (power demanded by the end-users). We embrace the Monte Carlo Simulation and Fuzzy Interval Analysis approach for the joint propagation of uncertainties (Li & Zio 2012, Baraldi & Zio 2008) considering the variations in time of random variables like solar irradiation and loads within a Functional Data Analysis framework where data are represented as functions of a continuous variable (e.g. time). This constitutes an innovative approach, as, traditionally, a unique probability density function is inferred from the historical data of one fixed period (e.g. a year), without considering that the data distribution evolves through time in a continuous way (Ramsay & Silverman 2005). Here, we wish to analyze that time variation, in order to i) find an estimate of a time-varying probabilistic model and ii) obtain more accurate results in the uncertainty analysis. As a quantitative indicator of the analysis we evaluate the Expected Energy Not Supplied, a reliability index commonly used in this field (Billinton & Allan 1996)

The results of the uncertainty propagation are compared with i) the pure probabilistic uncertainty propagation approach (Marseguerra & Zio 2002) and ii) the Monte Carlo Simulation and Fuzzy Interval Analysis approach considering the random variables constant in time, i.e. described by a unique probability density function.

The reminder of the paper is organized as follows. In Section 2, the case study is presented; in Section 3, the functional data analysis methods adopted for the modeling of time-varying data and the Monte Carlo and Fuzzy Interval Analysis approach used for the joint uncertainty propagation are detailed, in Section 4, the results are reported and commented; finally, in Section 5, conclusions are provided.

2 CASE STUDY

The case study concerns the design of a solar panel that provides electrical energy to a house located in the south of Spain. The size of the panel is a trade-off between its performance to satisfy the demand of energy and the high costs of construction and maintenance. To perform this evaluation we consider the demand of power requested by the end-users and the possibility of storing the generated exceedance power in a battery that is necessary when the power from the solar is not sufficient (e.g. during cloudy days) or it is completely absent (e.g. during nights). This issue deals with a big amount of uncertainty due to the stochasticity of the behaviour of the end-users, the variability of the solar irradiation, the lack of knowledge about some operation parameters of the solar panel.

In Section 2.1, the description of the system model is provided and in Section 2.2, the uncertainty representation of its input variables is presented.

2.1 *Description of the system model*

The system consists of three different parts: the solar panel, the load and the battery, as illustrated in Figure 1.

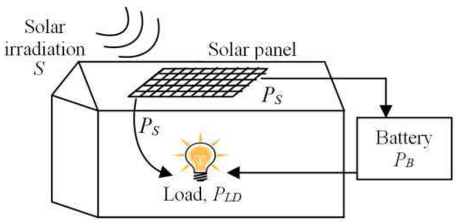

Figure 1. Scheme of the system

The power generated by the solar panel, P_S [kW], is a function of the solar irradiation, S, the number of solar cells, N, and a vector of operation parameters, $\boldsymbol{\theta}$ ($\boldsymbol{\theta} = I_{MPP}, V_{MPP}, V_{oc}, I_{sc}, N_{ot}, k_c, k_v, T_a$) (Li & Zio 2012):

$$P_S = N \cdot FF \cdot V_y \cdot I_y \tag{1}$$

where $I_y = S \cdot I_{SC} + k_c(T_c - 25)$, $V_y = V_{oc} - k_v \cdot T_c$, $T_c = T_a + S\dfrac{N_{ot} - 20}{0.8}$, $FF = \dfrac{V_{MPP} \cdot I_{MPP}}{V_{oc} \cdot I_{sc}}$.

I_{MPP} is the current at maximum power point [A], V_{MPP} is the voltage at maximum power point [V], V_{oc} is the open circuit voltage [V], I_{sc} is the short circuit current [A], N_{ot} is the nominal operating temperature [°C], k_c is the current temperature coefficient [A/°C], k_v is the voltage temperature coefficient [V/°C], T_a is the ambient temperature [°C].

The load, P_{LD} [kW], is the power demanded by the end-users.

The output model of the battery is the power, P_B [kW], that can be storage in the battery when the solar panel produces more power than the demand, i.e. when $P_S - P_{LD} = P_{Diff} > 0$, and can be given to the end-users when the opposite occurs, i.e. when $P_S - P_{LD} = P_{Diff} < 0$. In the present study we have adopted a dynamic model (Chen et al 2011) to represent the level of charge of the battery, calculating the difference between stored energies of two consecutive steps. The following equations describe the model of the battery when it is charging, i.e. $P_B(t) = -P_{Diff} < 0$, (see eq. 2 and 3), when it is discharging, i.e. $P_B(t) = -P_{Diff} > 0$, (see eq. 4 and 5) and when it is idle, i.e. $P_B(t) = P_{Diff} = 0$, (see eq. 6).

$$-\eta_c \cdot P_B(t) \cdot \Delta t_{\min} \le K_c Q_{\max} \tag{2}$$

$$Q(t+1) = Q(t) - \eta_c \cdot P_B(t) \cdot \Delta t_{\min} \tag{3}$$

$$P_B(t) \cdot \Delta t_{\min} / \eta_d \le K_d Q_{\max} \tag{4}$$

$$Q(t+1) = Q(t) - P_B(t) \cdot \Delta t_{\min} / \eta_d \tag{5}$$

$$Q(t+1) = Q(t) - W_{hourly} \tag{6}$$

where $Q(t)$ is the capacity of the battery at hour t [kWh], η_c and η_d are the charging and discharging efficiency, respectively, K_c and K_d are the maximum portion of rated capacity that can be added to and withdraw from storage in an hour, respectively, $Q_{\max}$ is the rated maximum stored energy, W_{hourly} is the battery hourly discharged energy [kWh], $\Delta t_{\min}$ is the scheduling interval. The parameter values adopted in the model are: $\eta_c = \eta_d = 0.85$, $K_c = K_d = 0.3$, $Q_{\max} = 40$, $W_{hourly} = 0.5$ kWh and $\Delta t_{\min} = 1$ hour. In this work, the initial level in the battery has been assumed to be equal to zero.

2.2 *Uncertainty representation*

In the model of the solar panel (eq. 1) the inputs can be classified in i) aleatory variable, i.e. the solar irradiation, ii) epistemic variables, i.e. the operation parameters of the vector $\boldsymbol{\theta}$, and iii) constant, i.e. the number of solar cells *N* that in the present simulation has been taken equal to 30.

The solar irradiation, *S*, [kW/m^2] depends on the variability of the weather. It is typically described by a probabilistic distribution, e.g. a Beta distribution, whose parameters, e.g. α and β, are inferred from sufficient historical data and are fixed for a given period (Li & Zio 2012). In the present paper, we still represent the solar irradiation with the Beta distribution but we consider its evolution in time estimating different values of the parameters α and β for each day of the year according the method explained in Section 3.1. The historical data used to infer the parameters are daily irradiation data in a geographical close area near Seville, Spain, (the square with latitude in the interval [37,38] and longitude in [-6,-5]), registered from July 1983 to June 2005 and stored in the database NASA: *Earth Surface Meteorology for Solar Energy* (NASA 2008). By way of example, Figure 2 shows an histogram of the historical data recorded and the correspondent Beta distribution of the solar irradiation in four different days of July (1st, 10th, 20th and 30th July, respectively).

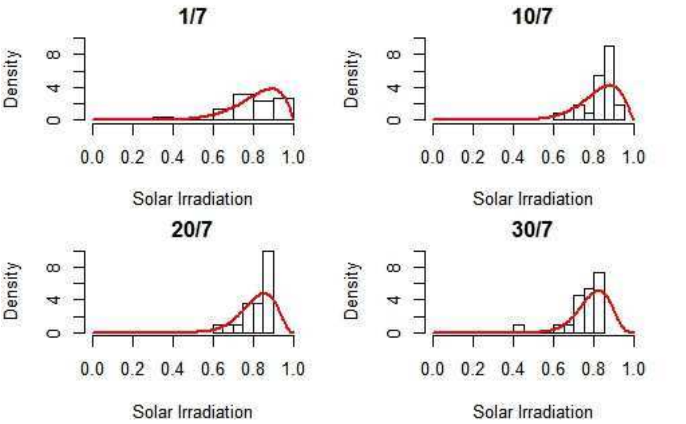

Figure 2. Histogram of the recorded data and the correspondent Beta distribution for the 1st, 10th, 20th and 30th July.

The operation parameters, $\boldsymbol{\theta}$, are classified into parameters provided by the manufacturers, e.g. $I_{MPP}, V_{MPP}, V_{oc}, I_{sc}, N_{ot}, k_c, k_v$, and by the end-users, e.g. T_a. Both are associated with epistemic uncertainty due to i) the lack of information provided by the manufacturers for commercial reasons and ii) the limited quantity of data available for each house for private issues (Li & Zio 2012). We represent these parameters by trapezoidal possibilistic distributions ($\pi^{I_{MPP}}$, $\pi^{V_{MPP}}, \pi^{V_{oc}}, \pi^{I_{sc}}, \pi^{N_{ot}}$, π^{k_c}, π^{k_v} π^{T_a}) as proposed in (Li & Zio 2012). In Figure 3, the possibilistic distribution $\pi^{N_{ot}}$ of the nominal operating temperature N_{ot} is reported. For the sake of brevity, we will not present here the basics of possibility theory; the interested reader can refer to (Dubois 2006).

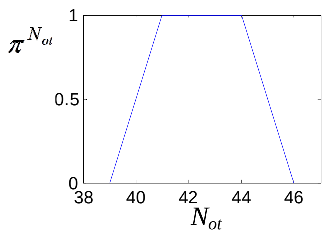

Figure 3. Trapezoidal possibility distribution, $\pi^{N_{ot}}$, of the nominal operating temperature, N_{ot}.

The load, P_{LD} [kW], is affected by aleatory uncertainty since its value depends on the behaviour of the end-users. Typically it is modelled by a normal probabilistic distribution (Liu et al 2011), with parameters inferred from the large amount of historical data available. In this work, we use a normal distribution, estimating two different time-varying mean values for days and nights, μ_{day}, and μ_{night}, respectively, following the procedure explained in Section 3.2, and maintaining a same standard deviation σ.

3 METHODS

In the following, the methods for the data analysis (Section 3.1 and 3.2) and for the joint uncertainty propagation (Section 3.3), are explained with reference to the model of Section 2.

3.1 *Uncertainty modeling of climatic data*

We observe $n=22$ realizations of irradiation data for the chosen location through time, during the year: for each time unit t_q (i.e., a day), we observe n different samples of data $S_i(t_q)$, where $i=1,\dots,n$ denotes the sample units, and $q=1,\dots,Q$ denotes the different time units. We suppose that, for a fixed time t_q, the observed data $S_i(t_q)$ is a random independent sample from a beta distribution of parameters $\alpha(t_q), \beta(t_q)$:

$$S_i(t_q) \sim Beta(\alpha(t_q), \beta(t_q)),\ \forall i=1,\dots,n,\ q=1,\dots,Q \qquad (7)$$

Finally, we assume that observations on different times are conditionally independent, given the values of $\alpha(t_q), \beta(t_q)$. In particular, this implies that the dependence structure of the solar irradiation on different days is entirely expressed by means of the time-varying structure of the two parameters, which we suppose can be modeled as smooth and regular functions of time, due to the intrinsic regularity of data.

To estimate the time-varying parameters we adopt the method of moment by expressing them in terms of moments of data for each time unit t_q, $q=1,\dots,Q$. We suppose that the moments of the distribution are regular one year-periodic functions and, since using the sample daily moments as estimates lead to extremely non-regular functions, we consider a method to regularize data, estimating a proper low dimensional functional space in which the moments are defined, by exploiting the procedure proposed in (Pini & Vantini 2013).

3.1.1 *Estimate of the mean function*

In order to estimate the mean function, we apply the Interval Testing Procedure (ITP) described in (Pini & Vantini 2013). The method consists in choosing a functional basis, calculating the basis expansion coefficients for each sample unit, and testing the significance of each basis function. The final result of the test is the selection of the basis components that are statistically significant to describe the mean function of data. As data are supposed to be *T*-periodic functions, with *T* equal to one year, a proper choice for the basis used to describe data is the *T*-periodic Fourier basis. Hence, we use an interpolating Fourier expansion:

$$s_i(t_q) = \frac{a_0}{2} + \sum_{k=1}^{(Q-1)/2} a_h \cos\left(\frac{2\pi}{T} h t_q\right) + b_h \sin\left(\frac{2\pi}{T} h t_q\right), \qquad (8)$$

which associates at each data and for each frequency h a bivariate vector of coefficients $(a_i^{(h)}, b_i^{(h)})$.

In order to test the significance of the basis functions, we perform the following bivariate test:

$$\begin{aligned} H_0^{(h)} &: \mathrm{E}\left[\left(\mathrm{a}^{(h)}, \mathrm{b}^{(h)}\right)^T\right] = (0,0)^T \\ H_1^{(h)} &: \mathrm{E}\left[\left(\mathrm{a}^{(h)}, \mathrm{b}^{(h)}\right)^T\right] \neq (0,0)^T \end{aligned} \quad . \qquad (9)$$

The test of equation 9 is performed by means of a permutation test (Pesarin & Salmaso 2010), based on the Hotelling T-square test statistic and on the joint permutations of the signs of the coefficients vectors.

Then, according to the ITP, we combine the results of the tests of eq. 9 by means of non parametric combination tests on all closed intervals, correcting the marginal p-values and providing an interval-wise control of the Family Wise Error Rate. Finally, we select as significant all the frequencies with an associated corrected p-value greater than 5%.

The estimate of the function will be the *T*-periodic function obtained by means of the Fourier expansion of the sample mean coefficients restricted to the selected frequencies. That is, if *v* is the vector identifying the final selection of significant frequencies, ($v_h = 0$ if the result of the *h*-th test is $H_0^{(h)}$, $v_h = 1$ if the result of the *h*-the test is $H_1^{(h)}$), the final estimate of the mean function is given by:

$$\hat{\mu}_s(t_q) = \frac{\bar{a}^{(0)}}{2} + \sum_{h \in v} \bar{a}^{(h)} \cos\left(\frac{2\pi}{T} h t_q\right) + \bar{b}^{(h)} \sin\left(\frac{2\pi}{T} h t_q\right) \quad (10)$$

3.1.2 *Estimate of the variance function*

The method used to find the estimate of the variance function is formally the same as the method described to estimate the mean. The only difference is constituted by the starting point of the test. In fact, the application of the ITP briefly described in the last paragraph enables to estimate the mean function of a set of functional data. Consequently, in order to find an estimate for an higher order central moment, we can associate to each sample unit another functional data $\delta_i^2(t_q)$, $i = 1,\ldots,n$, corresponding to the squared deviation from the mean:

$$\delta_i^2(t_q) = \left(s_i(t_q) - \hat{\mu}_s(t_q)\right)^2 . \quad (11)$$

Then, the variance of the original data, which is defined as the expected value of the deviations expressed in eq. 11, is estimated with the same method described above applied to the new data set composed by the functions $\delta_i^2(t_q)$.

3.2 *Uncertainty modeling of the load data*

As well as the solar irradiation, also the load P_{LD} [kW] has a time varying structure. In particular, we suppose that, for each day of the year t_q, the load has two normal distributions for days and nights, with the same constant standard deviation (σ = 0.25 [kW]) and two time-varying means ($\mu_{P_{LD},day}(t_q)$ and $\mu_{P_{LD},night}(t_q)$, respectively). The model assumed for the day and night load, for each time t_q is then the following:

$$P_{LD,day/night}(t_q) \sim N(\mu_{P_{LD},day/night}(t_q), \sigma^2), \ \forall q = 1,\ldots,Q . \quad (12)$$

To estimate the day and night mean functions, it is not possible to proceed applying the ITP to the daily load data, because in this case data are not directly available. We know that the daily mean electrical consumption of a house in the south of Spain is about 24.54 kWh (Sech-Spahousec 2011) and in the night the demand of electricity is the half than during the day (Omnie 2012). Thus, it is possible to infer that the means of the hourly load for days and nights are 1.363 kW and 0.682 kW, respectively. Since these data are aggregated through the entire year, it is not possible to infer a time varying distribution. Consequently, a different approach is here necessary.

We assume that most of the usual household electrical devices (e.g. washing machine, refrigerator, TV) are approximately used in the same way in summer and winter, and, thus, the electrical consumption of these devices is constant throughout the year. The only devices that may have a time-varying load are the air conditioning systems (whose load varies in the warm months depending on the external temperature) and the lighting (whose load changes through the year depending on the variation of daylight time). Since for the former, the load is higher than for the latter, we consider only the air conditioning systems (AC) as a device with a time varying load.

Starting from the daily minimum and maximum temperatures in the Seville area, stored in the NASA data base (NASA 2008), we calculate the time-varying mean of the load of an AC with some fixed characteristics. We consider a class "A" device, with an Energy Efficiency Ratio (EER) equal to 3.5. The number of AC installed in the house is set equal to the mean number of conditioners in Spanish homes in Andalusia, that is 1.623 (INE 2008). The nominal power of the AC is calculated as $P_N^{AC} = Surf \times Ceiling \times 25$ (ENEA 2006), where *Surf*=20m^2 is the surface of the room and *Ceiling*=2.7m is the higher of the ceiling. All data are chosen to indicate a representative Spanish house. Finally, since the proportion of Spanish that leave the AC turned on at night is equal to 7.6% (INE 2008), we multiply the AC load at nights by this proportion.

In order to calculate the mean load of such AC system, first of all, we find functional estimates for the mean tendency of the daily minimum and maximum temperature for the given location ($T_{min}(t_q)$ and $T_{max}(t_q)$ [°C], respectively), by means of the HP-Test on min and max temperatures, as shown in the procedure of Section 3.1.1. Then, for each day t_q, we perform the following calculation:

- We fix a threshold temperature T_{thres}=26°C, and suppose that the AC is turned on when the external temperatures exceed the threshold, as in (Izquierdo et al 2011).
- We estimate the daily lapse of time in which the AC is turned on $h_{on}(t_q)$ [h], supposing for each day a linear temperature profile between $T_{min}(t_q)$ and $T_{max}(t_q)$:

$$h_{on}(t_q) = 24 \cdot \left(\frac{T_{\max}(t_q) - T_{thres}}{T_{\max}(t_q) - T_{\min}(t_q)} \right) \quad (13)$$

This approximation is justified by the comparison of our results with a daily temperature profile, that can be estimated from hourly data (Free Meteo 2012).

- The quantity $h_{on}(t_q)$ is then divided into daily (10.00 a.m. - 10.00 p.m.) and nightly (10.00 p.m. - 10.00 a.m.) hours of switching on ($h_{on}^{day}(t_q)$ and $h_{on}^{night}(t_q)$, respectively), considering that $T_{max}(t_q)$ is attained at 4.00 p.m. and $T_{min}(t_q)$ at 6.00 a.m. (Free Meteo 2012).
- The mean power load on days of the AC is then calculated as:

$$\mu_{P_{LD},day}(t_q) = P_N^{AC} n_{room} h_{on}^{day}(t_q) / (12EER) \quad (14)$$

The mean load on nights, is:

$$\mu_{P_{LD},night}(t_q) = P_N^{AC} n_{room} h_{on}^{night}(t_q) 0.076 / (12EER) \quad (15)$$

Note that both quantities are divided by 12[h] in order to found an estimate of the hourly power.

- The quantities of $\mu_{P_{LD},day}(t_q)$ and $\mu_{P_{LD},night}(t_q)$ are finally added to the day and night fixed averages (mean load without AC), calculated in order to maintain the values of 1.363 kW and 0.682 kW as yearly means.

3.3 *Propagation of aleatory and epistemic uncertainty in the model of an energy system made of a solar panel, a storage energy system and the loads*

The Monte Carlo Simulation and Fuzzy Interval Analysis approach (Baraldi & Zio 2008) has been adopted for the joint propagation of the aleatory and epistemic uncertainties of the model described in Section 2 to compute the Expected Energy Not Supplied (EENS) over a period of interest. The method is based on the combination of the Monte Carlo technique and the extension principle of fuzzy set theory by means of the following two main steps (Baudrit et al 2006):

i. repeated Monte Carlo sampling of the random variables to process aleatory uncertainty;

ii. fuzzy interval analysis to process epistemic uncertainty.

Since the analysis is time-varying, these two steps have to be repeated for all the period of interest. In particular, the following time steps have been considered:

- $\Delta t_{\min} = 1$ hour is the smallest time step of the system model. The total number of hours in the period of interest is defined by the variable N_{steps};
- $\Delta t_{\max} = 12$ hours is the time interval in which the power generated by the solar panel, P_S, and that demanded by the end-users, P_{LD}, can be considered constant. This assumption has been introduced to reduce the computational time of the simulation and to distinguish only between day and night. Therefore, the total number of different values considered for those variables is $N_{steps} / \Delta t_{\max}$.

The operative steps of the procedure to compute the EENS index are the following:

1. Set $k = 1$ (outer loop processing aleatory uncertainty).
2. Sample the vector $\tilde{s}_l^k$, $l = 1, ..., N_{steps} / \Delta t_{\max}$, of the solar irradiation S from the Beta distribution (eq. 7) when l is an odd number (i.e. when it is day), otherwise, set $\tilde{s}_l^k = 0$ (i.e. when it is night). Then, sample the vector $\tilde{\boldsymbol{p}}_{\boldsymbol{LD}\,l}^{\,k}$, $l = 1, ..., N_{steps} / \Delta t_{\max}$, of the load P_{LD}, from eq. 12 taking into account the different distributions associated with that variable during the days and nights. The vectors $\tilde{s}_l^k$ and $\tilde{\boldsymbol{p}}_{\boldsymbol{LD}\,l}^{\,k}$, $l = 1, ..., N_{steps} / \Delta t_{\max}$, are transformed into $\boldsymbol{s}_j^k$ and $\boldsymbol{p}_{\boldsymbol{LD}\,j}^{\,k}$, $j = 1, ..., N_{steps}$, respectively, repeating each value $\Delta t_{\max}$ times, to obtain values of solar irradiations and loads for each hour in all the period of interest.
3. Set $\alpha = 0$ (middle loop processing epistemic uncertainty).
4. Set $j = 1$ (inner loop processing the time variation).
5. Select the corresponding α-cuts of the possibility distributions ($\pi^{I_{MPP}}, \pi^{V_{MPP}}, \pi^{V_{oc}}, \pi^{I_{sc}}, \pi^{N_{ot}}, \pi^{k_c}, \pi^{k_v}, \pi^{T_a}$) as intervals of possible values of the possibilistic variables $I_{MPP}, V_{MPP}, V_{oc}, I_{sc}, N_{ot}$, k_c, k_v, T_a.
6. Calculate the smallest and largest values of the solar power generated, $\underline{P}_{S\,j,\alpha}^{\,k}$ and $\overline{P}_{S\,j,\alpha}^{\,k}$, respectively, by eq. 1 considering the fixed value s_j^k sampled for the random variables S and all values of the possibilistic variables $I_{MPP}, V_{MPP}, V_{oc}, I_{sc}, N_{ot}, k_c, k_v, T_a$ in the α-cuts of their possibility distributions ($\pi^{I_{MPP}}, \pi^{V_{MPP}}, \pi^{V_{oc}}$, $\pi^{I_{sc}}, \pi^{N_{ot}}, \pi^{k_c}, \pi^{k_v}$ π^{T_a}).
7. Compute the value $\underline{P}_{Diff\,j,\alpha}^{\,k} = \underline{P}_{S\,j,\alpha}^{\,k} - P_{j,k}^L$: if $\underline{P}_{Diff\,j,\alpha}^{\,k} > 0$, go to step 7.a.; if $\underline{P}_{Diff\,j,\alpha}^{\,k} < 0$ go to step 7.b. else go to step 7.c.:
   a. set the Energy Not Supplied index equal to zero, $\overline{ENS}_{j,\alpha}^k = 0$, and increase the level of energy in the battery by eq. 3, $\underline{Q}_{j+1,\alpha}^k = f(Q_{j,\alpha}^k, \underline{P}_{B\,j,\alpha}^{\,k}, \eta_c)$, where $\underline{P}_{B\,j,\alpha}^{\,k} = -\underline{P}_{Diff\,j,\alpha}^{\,k}$ if the constraint defined in eq. 2 is verified, otherwise $\underline{P}_{B\,j,\alpha}^{\,k}$ is computed by eq. 2. If the level of energy in the battery at the step $j+1$ is higher than its maxi-

mum capacity, i.e. $\underline{Q}^k_{j+1,\alpha} > Q_{\max}$, then, set $\underline{Q}^k_{j+1,\alpha} = Q_{\max}$.

b. decrease the level of energy in the battery by eq. 5, $\underline{Q}^k_{j+1,\alpha} = f(\underline{Q}^k_{j,\alpha}, \underline{P}_{B\,j,\alpha}^{\;k}, \eta_d)$; if the constraint defined in eq. 4 is verified $\underline{P}_{B\,j,\alpha}^{\;k} = -\underline{P}_{Diff\;j,\alpha}^{\;\;k}$ (case i.), otherwise $\underline{P}_{B\,j,\alpha}^{\;k}$ is computed by eq. 4 (case ii.). If the level of energy in the battery at the step $j+1$ is higher than zero, i.e. $\underline{Q}^k_{j+1,\alpha} > 0$, compute the Energy Not Supplied index as $\overline{ENS}^k_{j,\alpha} = 0$, for the case i. and, $\overline{ENS}^k_{j,\alpha} = -\underline{P}_{Diff\;j,\alpha}^{\;\;k} - \underline{P}_{B\,j,\alpha}^{\;k}$ for the case ii.; otherwise, set, $\underline{Q}^k_{j+1,\alpha} = 0$ and $\overline{ENS}^k_{j,\alpha} = -\underline{P}_{Diff\;j,\alpha}^{\;\;k}$.

c. set $\overline{ENS}^k_{j,\alpha} = 0$, and decrease the level of the battery by eq. 6, $\underline{Q}^k_{j+1,\alpha} = f(\underline{Q}^k_{j,\alpha}, W_{hourly})$. If the level of energy in the battery at the step $j+1$ is lower than its minimum zero, i.e. $\underline{Q}^k_{j+1,\alpha} < 0$, then set, $\underline{Q}^k_{j+1,\alpha} = 0$.

8. Repeat steps 7. for the evaluation of the lower bounds of $\underline{ENS}^k_{j,\alpha}$, computing the upper values of $\overline{P}_{Diff\;j,\alpha}^{\;\;k}$, $\overline{P}_{B\,j,\alpha}^{\;k}$ and $\overline{Q}^k_{j+1,\alpha}$.
9. If $j \le N_{steps}$, then set $j = j+1$ and return to step 5. above; otherwise go to step 10 below.
10. Compute the total lower and upper bound of the ENS in the period under analysis as $\underline{ENS}^k_{\alpha} = \sum_j^{N_{steps}} \underline{ENS}^k_{j,\alpha}$ and $\overline{ENS}^k_{\alpha} = \sum_j^{N_{steps}} \overline{ENS}^k_{j,\alpha}$; the lower and upper bound of the EENS, $\underline{EENS}^k_{\alpha}$ and $\overline{EENS}^k_{\alpha}$, is obtained by performing the mean of $\underline{ENS}^k_{\alpha}$ and $\overline{ENS}^k_{\alpha}$ respectively.
11. Take the extreme values, $\underline{EENS}^k_{\alpha}$ and $\overline{EENS}^k_{\alpha}$, found in 10. as the lower and upper limit of the α-cut of the Expected Energy Not Supplied.
12. If $\alpha \neq 1$, then set $\alpha = \alpha + \Delta\alpha$ and return to step 4. above to compute the EENS for another α-cut; otherwise a fuzzy random realization, π^k_{EENS}, of the EENS has been identified. If $k \neq m$, where m is the number of simulations, then set $k = k+1$ and return to step 2. above; else stop the algorithm.

At the end of the procedure the fuzzy random realization (fuzzy interval) π^k_{EENS}, $k = 1, 2, ..., m$ of the Expected Energy Not Supplied index is constructed as the collection of the values $\underline{EENS}^k_{\alpha}$ and $\overline{EENS}^k_{\alpha}$, $k = 1, 2, ..., m$, found at step 10. above (in other words, π^k_{EENS} is defined by all its α-cut intervals [$\underline{EENS}^k_{\alpha}, \overline{EENS}^k_{\alpha}$].

On the basis of the rule of the possibility theory (Baudrit et al 2006), these possibilistic distributions can be aggregated. As a result, two cumulative distribution functions (cdfs), called belief and plausibility (i.e. the lower and upper cdfs, respectively), of the Expected Energy Not Supplied are obtained. They can be interpreted as bounding cumulative distribution functions (Baudrit et al 2006) and they contain all the possible cumulative distribution functions that can be generated by a pure probabilistic approach that considers all the inputs variables as probabilistic. For the sake of comparison, we have embraced also this method with m = 10000 samplings of the probabilistic variables: in this case, the possibilistic distributions of the input variables are transformed into probabilistic distributions by the normalization method given in (Flage et al 2008).

4 RESULTS

In this Section the results of the methods described in Section 3 are illustrated with reference to the case study of Section 2.

The top panel of Figure 4 shows the solar irradiation data in southern Spain (gray lines) and the estimates of the mean (red line) and variance functions (black lines), represented as the mean$\pm$two standard deviations, carried out with the method of Section 3.1. The Interval Testing Procedure (ITP) on Spain irradiation data selects as significant the mean value and the first three frequencies (the sinusoids and cosinusoids of period one year, six and three months) both for the mean and the variance functions. The final estimates are periodic functions fully described by the sample means coefficients: $(\overline{a}^{(0)}, \overline{a}^{(1)}, \overline{a}^{(2)}, \overline{a}^{(3)}, \overline{b}^{(1)}, \overline{b}^{(2)}, \overline{b}^{(3)})$.

The panels in the lower part of Figure 4 illustrate the comparison between the estimates of the mean and variance functions carried out with the method based on the ITP and with the one based on a daily estimate of mean and variance, respectively. It can be seen that the first method gives smooth curve, which follows the yearly fluctuations of the quantity of interest, whereas the second one gives extremely irregular functions.

The results of the analysis on the min-max temperature data, and the subsequent results of the load parameters are presented in Figure 5. On the top, the daily minimum and maximum temperatures data in southern Spain (light blue and red lines, respectively), are shown together with the ITP estimates of the two means. In this case, the ITP selects as significant the mean value and the first two frequencies both for the min and the max temperatures. The horizontal line indicates the threshold temperature at which the AC is turned on, T_{thres}=26°C. On the bottom panel, the estimates of the time-varying means of the load, for days and nights (yellow and black lines, respectively) are reported. Moreover, the Figure indicates the densities of the simulated day and night load $P_{LD}{}^{day}(t_q)$ and $P_{LD}{}^{night}(t_q)$ for a summer and a winter day.

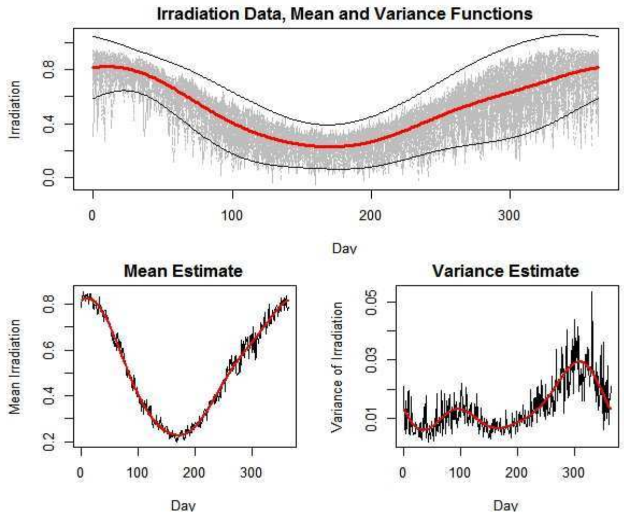

Figure 4. Top: Solar Irradiation Data in Southern Spain (gray), mean function estimate (red), mean ± 2*standard deviation (black). Bottom left: Comparison Between the Mean Estimate and the Daily Mean. Bottom right: Comparison Between the Variance Estimate and the Daily Variance

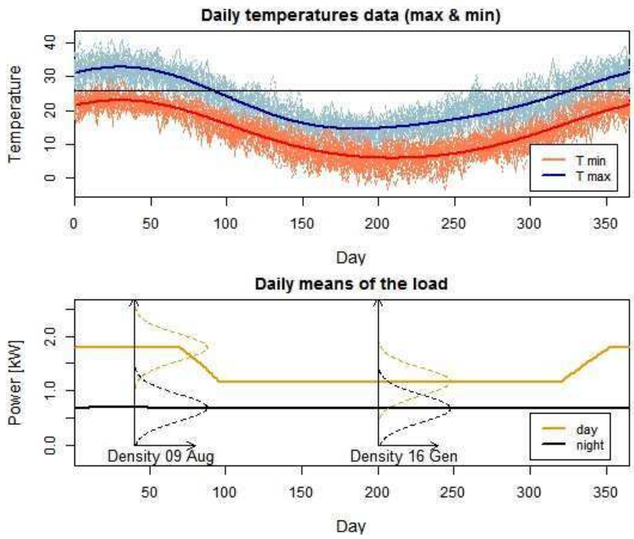

Figure 5. Top: daily min (light blue) and max (light red) temperatures data and ITP estimates for the means (bold blue and red lines); Bottom: estimates of the time-varying means of the load for days (yellow) and nights (black), and densities of simulated data in a summer and winter day.

The computation of the EENS index has been performed by applying the method of Section 3.3 considering the time-varying parameters of the probabilistic distribution of the solar irradiation and of the loads determined above. The analysis has been carried out with respect to the month of July that is a critical period for the high demand of power by the end-users. In fact, the hot temperature reached in the south of Spain gives rise to a large use of air conditioners.

Figure 6 reports the comparison of the cumulative distribution functions of the EENS index obtained by the probabilistic uncertainty propagation approach (solid lines) with the belief (lower curves) and plausibility (upper curves) functions obtained by the Monte Carlo and Fuzzy Interval Analysis approach described in Section 3.3.

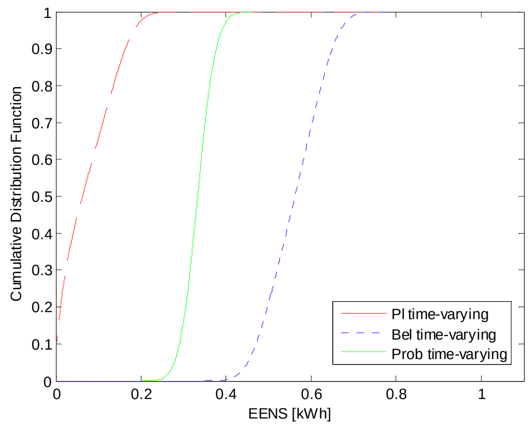

Figure 6. Comparison of the cumulative distribution functions of the EENS [kWh], obtained by the pure probabilistic propagation approach (solid line) with the belief (dotted line) and plausibility (dashed line) functions obtained by the Monte Carlo and Fuzzy Interval Analysis approach

The Monte Carlo and Fuzzy Interval Analysis method explicitly propagates the aleatory and epistemic uncertainty: the separation between the belief and plausibility functions reflects the imprecision in the knowledge of the possibilistic variables ($I_{MPP}, V_{MPP}, V_{oc}, I_{sc}, N_{ot}, k_c, k_v, T_a$) and the slope pictures the variability of the probabilistic variable (S and P_{LD}). Instead, the uncertainty in the output distribution of the pure probabilistic approach is given only by the slope of the cumulative distribution. As expected, the cumulative distribution of the EENS obtained by the pure probabilistic method is within the belief and plausibility functions obtained by the Monte Carlo and Fuzzy Interval Analysis approach.

Figure 7 compare the previous results, carried out with the Monte Carlo and Fuzzy Interval Analysis approach, with those obtained by the same method but by considering constant the parameters of the probabilistic distributions of the solar irradiation, S, and the loads, P_{LD}. For the comparison, the values of the 99th percentile are also indicated in the Figure.

It can be seen that:

- The lower and upper cumulative distributions functions obtained by considering time-varying parameters are always lower than those resulted by keeping constant those parameters. This means that a time-varying analysis allows designing the solar panel with smaller dimension.
- The gap between the cumulative distributions functions obtained by considering time-varying parameters is higher than that between the curves obtained by keeping constant those parameters. In particular, by considering time-varying parameters, we introduce a higher variability on the

EENS estimation, due to the fact that the distribution of data change daily. The higher variability allows considering within our model the situation in which the solar panel fully support the load demand, including the zero value in the EENS distribution.

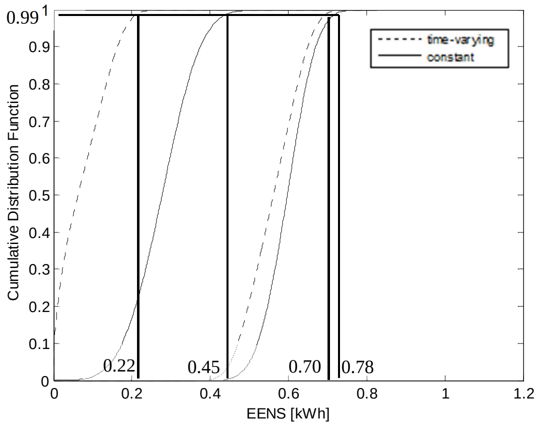

Figure 7. Left: Comparison of the lower and upper cumulative distribution functions of the EENS obtained by the Monte Carlo and Fuzzy Interval Analysis approach considering constant (solid line) and time-varying (dotted line) the parameters of the probabilistic distribution. Right: Lower and upper values of the 99th percentile.

5 CONCLUSIONS

We have applied to the model of an energy system made of a solar panel, a storage energy system and the loads, the Monte Carlo Simulation and Fuzzy Interval Analysis approach for the joint propagation of uncertainty to compute the Expected Energy Not Supplied (EENS) index. In particular, we have considered the variations in time of the random variables like solar irradiation and loads, describing them by probabilistic distributions with time-varying parameters.

Two main results have to be highlighted:

- the uncertainty propagation method divides the contribution of the aleatory and epistemic uncertainty, identifying an upper and a lower bound of values for the EENS, i.e. an interval of values of the EENS index is determined. This can be of interest in the decision making process to identify the proper size of the solar panel;
- Accounting for the time-varying parameters in the distributions of the solar irradiation and of the loads leads to more accurate results that let reduce the dimension of the solar panel. Thus, considering constant parameters an overestimation of the size of the solar panel can be done.

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